WARNER

The Determination of Dielectric Constants by a Resonance Method

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THE DETERMINATION OF DIELECTRIC CONSTANTS BY A RESONANCE METHOD

BY

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

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ENTITLED THE DETERMINATION OF DIELECTRIC CONSTANTS BY A RESONANCE METHOD

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF ARTS

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PART I HISTORICAL

I INTRODUCTION

The subject of dielectric constants has been a live interesting topic ever since 1748 when Benjamin Franklin[1] proved, by his dissectable Leyden jar experiment, that the energy of a charged condenser resided in the medium between the conducting surfaces. The next question asked was, would the nature of the medium change the amount of the energy stored up? Faraday proved that it did. For a term, to show the quantitative measure of this dependence upon the medium, he used "the specific inductive capacity" and defined it as the ratio of the capacity of a condenser with the given substance as the dielectric to the capacity of the same condenser with air as the dielectric. This name has become antiquated and now the term "dielectric constant" is generally used in its place.

Faraday[2] explained the laws of electrostatics by assuming the existance of "lines of force" throughout the medium surrounding charged bodies. He considered these lines as starting from positively charged bodies and ending on negatively charged bodies. He contrasted them to elastic strings, for he thought of them as always tending to shorten and therefore tending to bring the opposite charges at their ends nearer together. They were different from elastic strings in that they repelled each other. To explain the presence of these lines he considered the dielectric as being composed of small conducting particles imbedded in the nonconducting medium. When a condenser was charged he pictured these conducting particles as all being turned in one direction, that is polarized (as in Ewing's theory of magnetism.) Upon the discharge of the condenser the particles

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1 Benjamin Franklin, Letters on Electricity.

2 Michael Faraday, Experimental Researches, Vol. I, Sec. [illegible].

would resume their original position.

This theory was improved and strengthened by mathematical investigation by Mosotti and the result is now known as the Clausius-Mosotti[3] theory.

Faraday's theory was further improved by Maxwell[4] and later by J. J. Thomson[5]. Supposing the "lines of force" had definite volume the name "tubes" was substituted for "lines". It was supposed that each tube started from a unit positive charge and ended on a unit negative charge. By mathematical deductions Maxwell derived a relation between the dielectric constant, k, and the index of refraction, n, of a substance, namely

$$k = n^2 \tag{1}$$

According to Maxwell's and Thomson's derivation (1) should hold for any frequency. k and n should however be measured for waves of the same frequency. Many dielectric constant values have been obtained with constant or slowly alternating electric forces. These values show wide discrepancies from this so-called Maxwell's Law. To best check this law dielectric constants should be measured with very short electric waves, that is, with very high frequency electro-motive forces.

The theory which now receives the greatest approval is the electronic theory of H. A. Lorentz. Dielectrics are characterized by the fact that the electrons, which accompany every molecule, are prevented from leaving the molecules by the forces which act upon them.

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3 Clausius "Mech. Wärmetheorie", Vol. 2, p. 94, (1874).

4 J. C. Maxwell, Electricity and Magnetism, Vol. 2, p. 175, etc.

5 Sir J. J. Thomson, "Recent Researches in Electricity and Magnetism."

When a piece of a nonconductor is acted upon by no external charges the electrons arrange themselves with respect to the molecules so that there will be no external electro-static forces. When the nonconductor is brought between charged plates each electron will be displaced a small amount toward the positive plate, leaving the remaining portion of the molecule positively charged. From this theory it can be proven mathematically that

$$k = n^2$$

only when k is determined with constant or slowly alternating electric forces and where n is the index of refraction for infinitely long waves.

It is the purpose of this investigation to develop a method by which dielectric constants can be measured using high frequency alternations.

II ESSENTIALS OF A GOOD METHOD

A good method for determining dielectric constants will combine accuracy with ease and rapidity; it will not require large amounts of the material to be measured; it will not require that the dimensions of the material be known; the labour of computation must be a minimum; it must be possible to determine approximately the frequency of the alternations; and the arrangement should be such that it would be possible to study the dielectric under different conditions of temperature and pressure.

III J. J. THOMSON'S METHOD[6]

J. J. Thomson was one of the first to measure dielectric constants with rapidly alternating forces. His apparatus is shown diagrammatically in Figure I.

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6 J. J. Thomson, Proc. Roy. Soc., 46, p. 292, (1889).

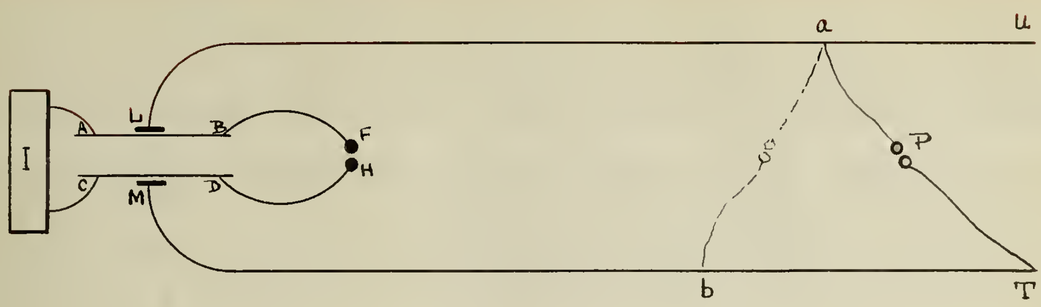

Fig. 1

AB and CD were the plates of a condenser, each being 30 cm. in diameter. They were connected to an induction coil and also to the spark gap FH. L and M were small plates placed very close to the condenser plates. From these two plates long thin parallel wires LU and MT extended for 20 meters. When the induction coil was started sparks occurred at FH and the system oscillated with its own frequency given by the formula

$$T = 2\pi\sqrt{LC} \qquad (1)$$

where L and C are the inductance and capacity of the oscillating system, and where the resistance is negligible. The impulses which were in the condenser sent electric waves down the two wires. These waves would be reflected and advancing waves would interfere causing points of maximum and minimum potential along the wire. The substance whose dielectric constant was to be measured (glass) was placed between the condenser plates and the induction coil started. The wave length of the oscillating system was determined as follows:- two equally long wires were connected to a spark micrometer P. The free ends were then connected to T and U. The contact at U was moved out to some point a where the sparks in the micrometer were a minimum - showing that T and a were at the same potential. Then the contact at T was moved out to b where the sparking in the micrometer was again a minimum,

showing that a and b were at the same potential. Then b and T were at the same potential and since T is a point of maximum potential bT was a wave length. Knowing the wave length λ and substituting for T its value $\frac{\lambda}{v}$, where v is the velocity of propagation of electromagnetic waves, into (1) we have

$$T = \frac{\lambda}{v} = 2\pi\sqrt{LC_x} \qquad (2)$$

From this equation C_x, the capacity of the condenser with glass as the dielectric could be computed. Everything was then known except L, the inductance of the circuit and this was computed from a formula. C_x divided by C_a, the capacity of the condenser with air as the dielectric, gives the dielectric constant k, i.e.

$$k = \frac{C_x}{C_a} \qquad (3)$$

C_a was computed from the formula

$$C_a = \frac{A}{4\pi d} \qquad (4)$$

where A is the area of one of the plates and d the distance between them. Allowance was made for the condenser having some of its capacity due to other conductors in the field. Thomson computed his frequency to be about twenty-five million (25,000,000).

It can be seen that the measured wave length was squared in order to solve (2) for C_x. Any per cent error in λ was therefore doubled in the result. To obtain bT, the wave length, two minimum sparking points in the spark micrometer were observed. Experience shows it to be very difficult to use a spark micrometer with accuracy.

IV C. B. THWING'S METHOD[7]

Thwing was the first to determine dielectric constants by a

7 Zeit. Phy. Chem. 14, p. 286, (1894) or
Phys. Rev., 2, p. 35, (1894-95).

resonance method. He states that the idea was suggested to him by Professor Hertz under whom he was working. The apparatus with which he worked is shown in Fig. 2.

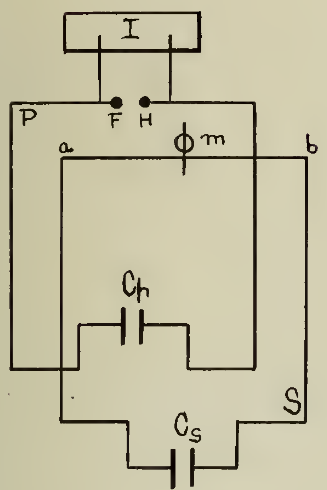

Fig. II

The circuit P, the primary, consisted simply in a rectangle of small wire, about 60 cm. square, in which there was a spark gap and a variable air condenser C_p. The induction coil I was connected to the primary as is shown. When the coil was in operation sparks jumped across FH and the primary oscillated with a period determined by the inductance, capacity and resistance of the circuit. In order that the secondary S, which was of the same dimensions as the primary and placed about 15 cm. distant, be in tune with the primary, the following equation must be true.

$$T = 2\pi\sqrt{LC_s} \tag{5}$$

where T is the period of both the primary and secondary, L and C_s are the inductance and capacity of the secondary in Henries and Farads respectively. This assumes the resistance negligible. Evidently the maximum current will be produced in the secondary when it is in tune with the primary. In order that the two circuits be in tune the primary capacity C_p was varied until the current in the secondary was a maximum. This maximum current was determined by the dynamometer invented by Hertz. Its construction is shown in Fig. ~~3.~~ IV

The side ab was a thin German-silver wire divided in the middle and the two halves soldered to a small metal rod cd. cd was fastened

by thin steel wires, below to a stationary support e, and above to a torsion head T which was turned until the side ab was taunt and then

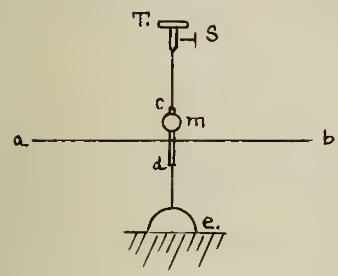

Fig. IV

it was set by the set screw S. Whenever a current was sent through ab it became heated and therefore expanded and the expansion turned cd around its vertical axis. A mirror was fastened rigidly to cd and the maximum current in the secondary was determined by observing a maximum deflection produced by means of a lamp and scale. Suppose the capacity in the secondary were C_a, C_a being the capacity of some condenser with air as the dielectric. When the coil was in operation the capacity in the primary was varied until a maximum deflection was observed. Then the period in both primary and secondary would be

$$T = 2\pi\sqrt{LC_a} \tag{6}$$

Then this condenser was taken out of the secondary and a variable parallel plate condenser was substituted and the distance between the plates was changed until the deflection was again a maximum. Suppose this capacity were C_1. Its value in C.G.S. units was computed from Kirchhoff's formula, namely

$$C = \frac{R^2}{4a} + \frac{R}{2\pi}\left(\log_\epsilon \frac{8\pi R}{a} - 1\right) \tag{7}$$

where R is the radius of the plates and a the distance between them. Then

$$T = 2\pi\sqrt{LC_1} \tag{8}$$

By comparing (6) and (8) it can be seen that

$$C_1 = C_a \tag{9}$$

In a similar way the capacity of the condenser with the unknown substance as the dielectric was found to be some value, say C_2. Then by definition of the dielectric constant, k would be

$$k = \frac{C_2}{C_1} \tag{10}$$

It is shown that when a spark jumps between two metal balls the resulting oscillatory current is by no means constant. Thwing says "the alternate heating and cooling of the wire produces small oscillations in the mirror, which while blurring the image to such an extent as to exclude the use of a reading telescope, are not sufficient to prevent accurate readings with a lamp and scale." If his dynamometer had been more sensitive this would not have been the case, so for refined measurements a modification is necessary.

V PAUL DRUDE'S METHOD[8]

P. Drude did an enormous amount of work on dielectric constants and one of his many similar methods is a resonance method. This is his so-called second method.

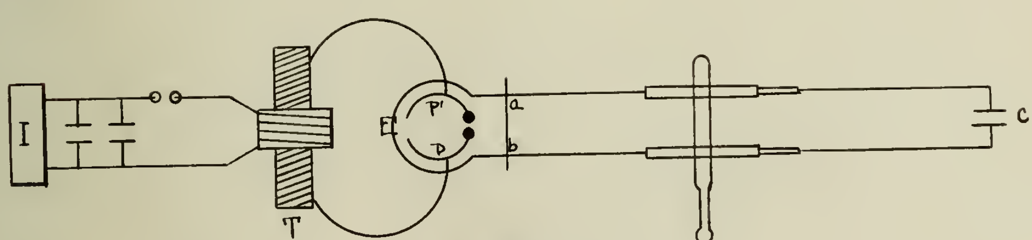

Fig. III

The high frequency current from a Tesla coil T, causes oscillations to be set up in the two semi-circular rods PP'. This induces an oscillation into the circuit aEb which was directly below PP' separated from it by mica, both circuits being immersed in kerosene. The

8 Zeit. Phys. Chem., 40, p. 635, (1902).

resonating circuit is acb and it is tuned with the primary by decreasing or increasing its inductance by pushing in or pulling out the telescoping tubes. The point of resonance was determined by the maximum glow of a Giesler tube placed between c and ab at a point of maximum potential. The capacity to be studied was c.

This method is similar to Thwing's with the exception that tuning is accomplished by varying the inductance instead of the capacity. In general the distance ac is not long and therefore the difficulty of obtaining it accurately brings a considerable per cent error into the result. Drude concluded that under the working conditions the error might be from 2 to 3%.

VI E. S. FERRY'S METHOD[9]

In 1897 E. S. Ferry devised a modification of Thwing's method, by which dielectric constants could be determined by a null method. His method consisted in getting two circuits of equal self inductance in resonance with a third oscillating circuit. The three circuits are shown in Figure 5.

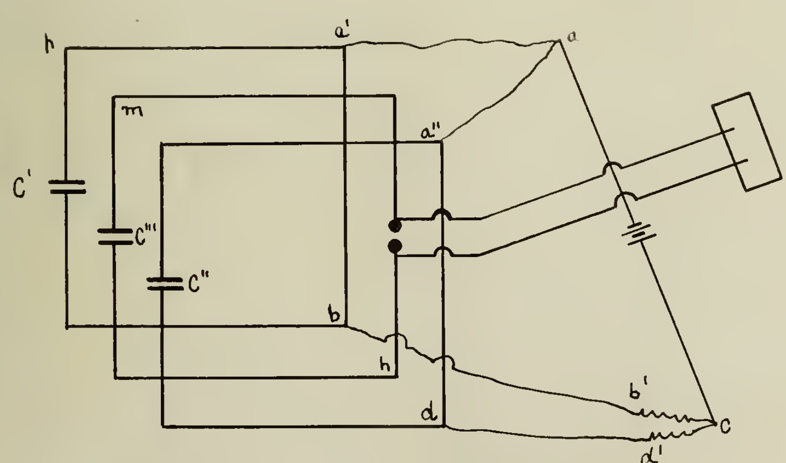

Fig. V

9 Phil. Mag., 5:44, p. 104, (1897).

The oscillating circuit was mh and the two resonating systems were pa'b and ga"d. When all three were in resonance the capacity C', which was a variable parallel plate condenser, in ph must equal the capacity C", which was the capacity under consideration with air as the dielectric, in gd because the period and inductance of the two circuits were the same. Thus C" could be determined by computing C'. Then when the dielectric to be measured was in C", changing its capacity to C_1'', and C' had been changed to C_1' in order that the three circuits be in resonance again, it could be said that C_1' (which was computed) was equal to C_1''. Then by definition

$$k = \frac{C_1'}{C'} \tag{11}$$

In theory this method seems simple for the working formula consists only in the ratio of two computed capacities but the method of tuning is not so simple. The principle of the bolometer was used to detect resonance. The application of this instrument can be shown in the following figure.

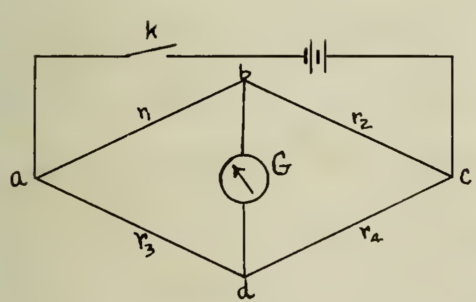

Fig. VI

The set up is the familiar Wheatstone Bridge set up. If r_2 equals r_4 the galvanometer will show no deflection when the key k is closed if r_1 equals r_3. If r_1 and r_3 are made of the same material, that is, have the same temperature resistance coefficient and are of equal resistance at one temperature they will be of equal resistance at all temperatures. Under these conditions if r_1 and r_3 are raised to any temperature the balance of the bridge is not disturbed as long as the rest of the bridge is kept under the

conditions
original. Referring again to Fig. 5, the side a'b of the secondary circuit pb was inserted as a part of the branch ab or r_1 (Fig. 6) and the side a"d of the other secondary circuit formed part of the branch ad or r_3 (Fig. 6). r_2 and r_4 were exactly equal resistances, coils in this case, b'c and d'c. First it was necessary to tune individually each of the secondary circuits with the primary. The condenser to be studied, with air as the dielectric, was placed in one of the secondary circuits, say pb, and the condenser removed from the other secondary. Then when the primary was set into oscillation the induced currents in pb heated ab, or part of r_1, Fig. 6, and the Wheatstone Bridge balance was disturbed and the galvanometer caused to deflect. When the two circuits were in tune, and tuning was accomplished by changing the capacity in the primary, the maximum current oscillated through ab and this caused a maximum deflection of the galvanometer. Then this condenser was removed from the secondary pb and placed in the secondary gd. Since the two secondaries have the same inductance the deflection in this case would also be a maximum, but in the opposite direction and perhaps not of the same magnitude because the two secondaries were not equi-distant from the primary. If this was the case gd was moved until this second maxima was just equal to the first. Then the variable parallel plate condenser was placed in the other secondary circuit (ph) and adjusted until the galvanometer showed no deflection. When this point was reached the two condensers in the secondary circuits could be interchanged and the deflection remain zero. The capacity to be measured is now equal to the capacity of the parallel plate condenser which could be computed. The above operations were repeated with the dielectric to be measured in the condenser. This operation gave the final data needed to substitute in equation 11.

It can be seen that the manipulation in this method is not so simple. Great care has also to be taken to protect the bolometer from temperature changes. Quoting from Ferry "all parts of the bolometer must be carefully screened from heating effects. Air draughts and similar sudden changes can be guarded against by thick coverings of cotton wool." While this method is a null method the zero deflection is produced by the effects of the two maxima counterbalancing each other, and each of the maxima had to be determined, in other words the errors in determining each remain in the result.

Ferry computed the frequency of his oscillations to be about 33,000,000 per second.

VII C. NIVEN'S METHOD[10]

Niven determined dielectric constants by a resonance method using a Fleming cymometer as an instrument to detect resonance. Thwing's arrangement was reversed and the capacity to be studied was put in the primary in series with inductance. The Fleming cymometer was the secondary. Instead of determining resonance by the maximum glow of a Neon tube the cymometer circuit contained a small coil, inside of which was a thermoelectric junction. The current from the junction caused a sensitive galvanometer to deflect. The set up is shown in Figure 7.

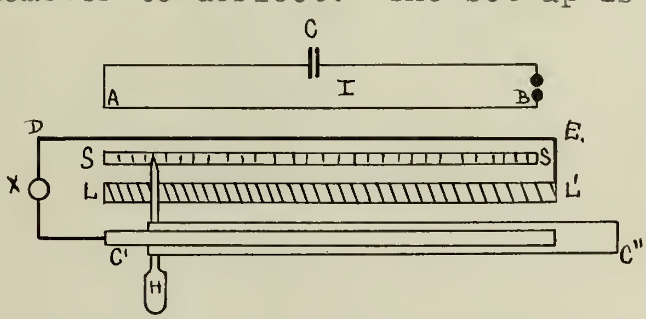

Fig. VII

10 Proc. Roy. Soc., 85, p. 139, (1911).

Circuit I is the primary with the capacity C and the inductance of the rectangular wire. The spark gap is excited by an induction coil. The secondary is the cynometer which consists of the wire C'XE in series with a variable inductance LL' and the variable tubular condenser C'C". When the handle H is shifted both the inductance and the capacity are changed. The oscillation in the cymometer heated the coil X and some of this heat was radiated to the thermo junction which was placed within X. For this experiment that particular scale was used which calibrated in terms of $\sqrt{CL}$. The condenser C was a spherical condenser of capacity, with air as the dielectric, of 17.8 c.g.s. units as computed by the formula

$$C = \frac{rr'}{r' - r} \qquad (12)$$

with water in C the cymometer tuned, that is, the galvanometer deflection was a maximum, when the scale reading was 14.5. k 17.8, where k is the dielectric constant of water, would be the capacity of C with the water in it and if T is the period

$$T = 2\pi\sqrt{L\ k\ 17.8} = 2\pi\ 14.5 \qquad (13)$$

Then an air leyden of computed capacity 1047 cm. was substituted for C and the cymometer tuned at 11.7. Then

$$T_1 = 2\pi\sqrt{L\ 1047} = 2\pi\ 11.7 \qquad (14)$$

Dividing (13) by (14) and squaring

$$\frac{k\ 17.8}{1047} = \left(\frac{14.5}{11.7}\right)^2 \qquad (15)$$

and solving for k

$$k = 90.36 \qquad (16)$$

This particular case for water shows how dielectric constants can be determined by using a cymometer.

Niven found that conducting liquids such as water, alcohol, etc.

would not permit a discharge to take place. To avoid this difficulty he put in series with C a condenser of large capacity. This forced the conducting capacity into oscillations while it did not change the resulting capacity of the primary. This can be seen to be true from the formula

$$c = \frac{C_1 \; C_2}{C_1 + C_2} \tag{17}$$

which gives the capacity of two condensers C_1 and C_2 when connected in series. If C_2 is very large compared to C_1 equation (17) becomes, to a very close approximation

$$C = C_1 \tag{18}$$

However, because of the large condenser more energy was used and the condenser was heated to a considerable extent. A constant temperature was maintained by allowing the liquid under consideration to continually flow through C, and also by immersing C in a large tank of water which could be kept at the desired temperature.

Fleming[11] has shown that in many cases the capacity measured in this way depended to a considerable extent upon the length of the spark gap in the primary. Anderson[12], working in this laboratory with a cymometer decided that 2.1% error in dielectric constant determinations was unavoidable by this method.

VIII HERMAN ROHMANN'S METHOD[13]

Rohmann developed a very interesting resonance method for studying the variation of the dielectric constant of gases with pressure.

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11 J. A. Fleming, Principles of Electric Wave Telegraphy and Telephony, p. 180.

12 S. H. Anderson, Phys. Rev., 34, p. 34, (1912).

13 Ann. d Phys., 4:34, p. 979, (1910-11).

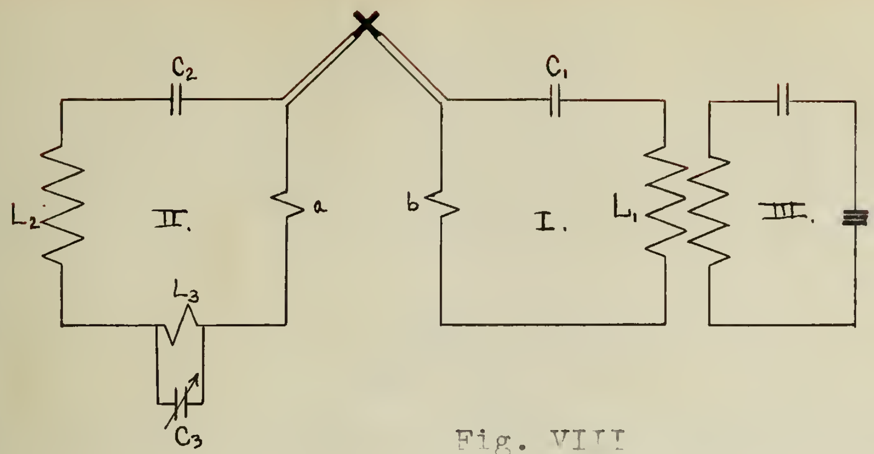


Fig. VIII

Circuit I is the primary and II the secondary. The oscillations in I are induced in it from the circuit III. It has been shown by Diekmann[14] that when the specially constructed dynamometer shows a zero deflection

$$C_1L_1 = C_sL_s \tag{19}$$

where C_1 and L_1 represent the capacity and inductance of circuit I and C_s and L_s are the total capacity and inductance of circuit II. Rohmann was able to study the Clausius-Mosotti relation as applied to gases without directly determining dielectric constants. He claims to be able to measure capacity changes to an accuracy of 1 in 100,000.

It is interesting to see if this accurate method can be extended to study substances which have dielectric constants greater than those of gases. His accuracy comes from the fact that the inductance L_3 is small compared with L_2. Suppose the circuits were in tune when the capacities in II were C_2 and C_3. Let C_2 be changed by an amount dc_2 and let C_3' represent the value of C_3 necessary for resonance. Then it can be shown, to a close approximation, that

14 M. Diekmann, Ann. d Phys., 24, p. 771, (1907).

$$\frac{dC_2}{C_3 - C_3'} = - \frac{L_3}{L_2}^2 \qquad (20)$$

Now if the inductances L_2 and L_3 were in such a ratio that the right hand member of (20) had a numerical value of .001 equation (20) would become

$$\frac{dC_2}{C_3 - C_3'} = -.001 \qquad (21)$$

Suppose the absolute value of $C_3 - C_3'$ were 10 cm., then

$$dC_2 = .01 \text{ cm.} \qquad (22)$$

Thus by this arrangement if C_3 could be changed by an amount of 10 cm., and practically this could be done very easily, it would be possible to measure a change of capacity in C_2 of .01 cm. This example shows how the accuracy was obtained. To apply this method to determine dielectric constants one of two plans could be used.

1 L_2 and L_3 must be known, as in the above example, and then the change of capacity, when the dielectric was added, could be computed and from this the dielectric constant determined.

2 C_2 could be changed by various known amounts and C_3 calibrated to read these changes.

Plan (1) does not seem feasible because of the difficulty of accurately determining small inductances. Any per cent error made in determining the small inductance L_3 is doubled in the result because L_3 is squared according to (20). Plan 2 is but slightly more favorable. C_3 would have to be calibrated against condensers placed in C_2 whose capacity could be computed. A guard ring could not be used and that means that the computed values might be in error as high as 1%. There is a further objection which applies to either plan. To measure a dielectric constant even as low as 2 means that C_3 would have to be

changed an enormous amount in order to offset the doubling of C_2 when the dielectric was placed between the plates. This is a case where the method of obtaining accuracy leads one to a design of apparatus which is impossible to obtain practically.

So while this method is a very accurate one to study gases, whose dielectric constants are low, it seems to be impractical for the study of substances which have higher dielectric constants.

The spark gap in circuit III was such as to produce a quenched spark. This has the great advantage of giving a constant uniform oscillation. It seems that this improvement could be applied with profit to any of the previously described methods.

PART II EXPERIMENTAL

I GENERAL DESCRIPTION OF THE METHOD

It was decided to try to develop an accurate method by modifying Thwing's method and to use as a detector of resonance a Duddell thermo-galvanometer. Figure 9 shows the essential features of the final arrangement of the apparatus. Circuit I, the primary, contains a capacity C_p and the inductance of a coil L_p. Circuit II, the secondary, contains the inductance L and a variable Korda condenser C. The thermo-galvanometer is shunted across the capacity as is shown. The primary oscillates with a definite period determined by its capacity, inductance and resistance. By varying the Korda C to some value, say C' the secondary will have the same period as the primary; that is, it will be in tune with it and then the maximum current will oscillate in

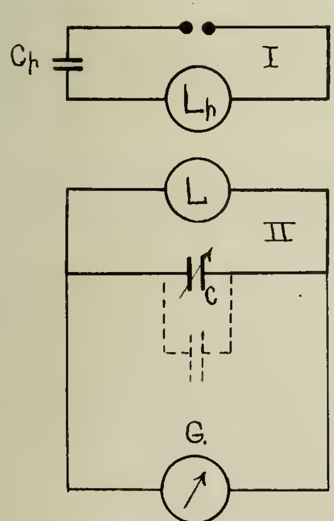


Fig. IX

the secondary, and the thermo-galvanometer will give a maximum deflection. Then

$$T = 2\pi\sqrt{LC'} \tag{23}$$

where T is the period of both the primary and secondary and L and C' the inductance and capacity of the secondary. Then the condenser under consideration, a conical condenser with air as the dielectric, was placed in parallel with the Korda as is shown by the dotted lines. When placed in parallel its capacity is added to the Korda, therefore to produce resonance the Korda had to be reduced to some value, say C". Then since the period is the same as before

$$T = 2\pi\sqrt{L(C''+C_a)} \tag{24}$$

where C_a is the capacity of the cone with air as the dielectric. Then the liquid to be studied was poured into the cone and the Korda tuned at, say C"'. Then

$$T = 2\pi\sqrt{L(C''' + C_x)} \tag{25}$$

where C_x is the capacity of the cone with the liquid being studied as the dielectric. By comparing (23) and (24)

$$C'' + C_a = C' \tag{26}$$

Therefore

$$C_a = C' - C'' \tag{27}$$

By comparing (23) and (25)

$$C''' + C_x = C' \tag{28}$$

Therefore

$$C_x = C' - C''' \tag{29}$$

Then by definition of the dielectric constant k

$$k = \frac{C_x}{C_a} = \frac{C' - C'''}{C - C''} \tag{30}$$

A calibration curve was plotted for the variable Korda C which gave its capacity in cm. for any reading of the scale. So the C's in the right hand member of the equation were obtained very easily. It was

found to be more accurate to use instead of the denominator of (30) C_1, the capacity of the cone with air as the dielectric as determined by the electrometer. Then the formula became

$$k = \frac{C' - C'''}{C_1} \tag{31}$$

In determining the dielectric constants of solids a slab of the solid was obtained and placed between two pieces of tin foil. The tin foil was kept close to the slab by the pressure of sheets of lead. The capacity of such a condenser was determined by the above method. The capacity of the condenser with air as the dielectric was determined from the formula

$$C = \frac{A}{4\pi d} \tag{32}$$

where A is the area of one of the sheets of tin foil and d the thickness of the slab. Then by definition the ratio of these two capacities gives the dielectric constant.

II DESCRIPTION OF THE APPARATUS

The Spark Gap.- When the spark was a simple one as is shown in Figure 9, the induced oscillations in the secondary varied greatly. This was shown by the galvanometer readings jumping back and forth so that the maxima could not be determined at all. Many combinations of spark gaps were tried. The most successful arrangement is shown in

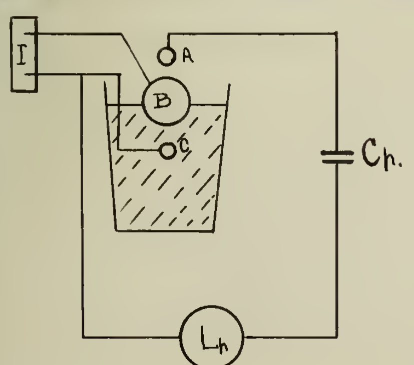


Fig. X

Fig. 10. The induction coil was connected to the zinc balls B and C. The spark between B and C occurred under kerosene. The capacity and inductance of the primary were connected as is shown. The oscillations of the primary

took place from A to B to C. The energy in the primary was so small that the discharge was in the form of a very faint glow between A and B. With this arrangement the induced currents in the secondary were nearly constant and therefore the galvanometer deflections were nearly constant.

The Galvanometer.- The galvanometer was a Duddell thermo-galvanometer. In principle it is very similar to Professor C. V. Boys radio micrometer. A loop of one turn, C, suspended by a fine quartz fiber, F, hangs between the poles of a permanent magnet. The loop is closed at the bottom by a thermal junction of Antimony Sb and Bismuth Bi. Just below the thermal couple is a small wire resistance which serves as a heater. The heat radiated from the heater, produced by the current in it, causes the thermo-couple to send a current through the loop C, and then it tends to place its plane perpendicular to the lines of force. Deflections were observed by means of a lamp and scale and the mirror m. This galvanometer seems to be one of the most sensitive instruments for detecting high frequency currents. For this experiment one could not ask for a more sensitive detector.

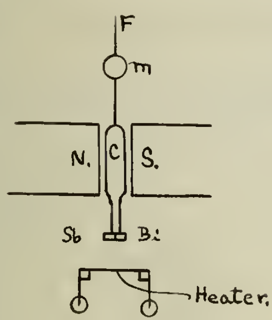
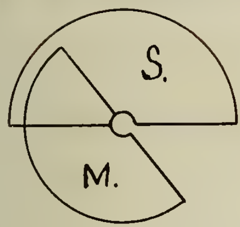


Fig. XI

Condensers.- The primary was a variable Korda[15] condenser. It consisted of two sets of semi-circular plates. The sixteen plates in the set S were connected together and held stationary, while the fifteen plates in the set M were connected together and arranged so that

15 Korda German Patent, No. 72447, Dec. 13, 1893.

they could be moved about a central axis. By rotating the moveable set M the area of the plates interlapping could be changed and thus



the capacity could be varied at will. The amount of the interlapping area could be read on a six inch circular scale, reading from 0° to 100°, placed on the box in which the plates were mounted.

The secondary variable condenser was a similar Korda with the exception that there were only eight fixed plates and seven moveable ones.

The Test Condenser.- The condenser in which the liquids were studied was a conical condenser similar to the one used by Fleming and Dewar[16]. Figure 12 shows a cross section.

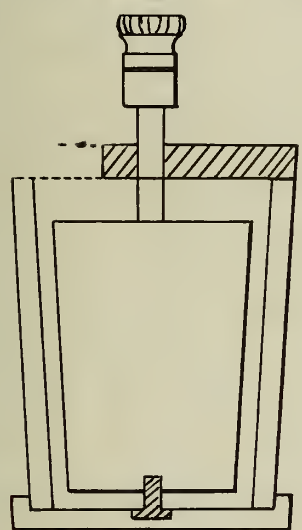


Fig. XII

The distance between the condenser walls was about 2.5 mm. and the taper was very slight. The cone was centered and held rigid at the top by a three legged ebonite spider and at the bottom by a small ebonite pin. The spider was fastened to the outside casing by three small screws. With the spider off and the bottom of the casing removed by unscrewing, the apparatus could be cleaned very easily. This was made of brass and plated with platinum so as to prevent corrosion.

The Inductances.- The primary inductance L_p was a coil of six turns of rubber covered copper wire 0.9 mm. in diameter. The coil was

16 J.A. Fleming and Dewar, Proc. Roy. Soc., Vol. 61, p. 279, (1897)

wound on a wood disc 14.5 cm. in diameter.

The secondary inductance L_s was simply one turn of the same size wire wound on a similar disc.

The Induction Coil.- The coil used was a Max Kohl 30 cm. induction coil. Such a large coil was not needed however, for only a small amount of energy was consumed.

III CALIBRATION OF THE CONDENSERS

The variable Korda condensers had such small capacities that they could not be calibrated by the ordinary method with a ballistic galvanometer, by comparing the quantity of electricity on them when raised to a given voltage to the quantity on a standard condenser when raised to the same voltage. So an electrometer method was resorted to.

The electrometer was set up in a large grounded iron box with a glass window. Within the iron casing it was protected from extraneous static effects. To be sure that the needle was placed symmetrical with respect to the quadrants it was adjusted so that when the needle was charged and the quadrants grounded the scale reading was just the same as when the needle and the quadrants were both grounded. Since the capacity of the electrometer was comparable with the capacities to be calibrated its capacity had to be determined.

This was done by a method of mixtures. The needle was charged to 50 volts, one pair of quadrants grounded and the other charged to a potential V (about 4 volts) causing a certain deflection, say d. Then the quantity of electricity Q, on the electrometer would be

$$Q = C_x V = C_x kd \qquad (33)$$

where C_x is the capacity of the electrometer and k is the constant of proportionality between the potential and the deflection. Then this charge was allowed to mix with the inside of a cylindrical condenser

of capacity C, while the outside was grounded, causing the deflection to reduce to d'. Then

$$Q = (C_x + C)V' = (C_x + C)kd' \tag{34}$$

From (33) and (34)

$$(C_x + C)d' = C_x d$$

which solving for C_x gives

$$C_x = C\frac{d'}{d-d'} \tag{35}$$

C, the capacity of the cylindrical condenser was computed from the formula

$$C = \frac{1}{2 \log_\epsilon \frac{r'}{r'}} \tag{36}$$

where r' is the inside radius of the outside cylinder and r is the outside radius of the inner cylinder.

The variable Korda was calibrated by mixing the quantity on the electrometer and two cylindrical condensers in parallel with the Korda set at every 10° position between 0° and 180°. The formula can be deduced in an exactly similar way to the one above. It is

$$C_k = C_1\frac{d-d'}{d'} \tag{37}$$

where C_1 is the combined capacity of the electrometer and the two cylindrical condensers, d the deflection when the electrometer and cylinders are charged and d' the deflection when the Korda had been added in parallel.

The capacity of the test condenser was also determined by this method. The charge on the electrometer was mixed with the cone and a cylindrical condenser connected in parallel. The formula in such an arrangement is (37) where C_1 is the capacity of the electrometer, d the deflection when it alone is charged, and d' the deflection when

the two condensers were added. Then the determined capacity minus the capacity of the cylinder gives the capacity of the cone.

All of the apparatus was placed in a grounded metal box to protect it from outside static effects. The connections were made by raising or lowering contacts into mercury cups. These mercury keys were operated by long silk threads so the body never came near any of the apparatus.

The calibration curve for one of the Korda's is shown in Fig. 13.

IV PLATINIZING THE CONDENSER

It was discovered that many liquids reacted chemically with the brass condenser and so it was decided to platinum plate the cone. The solution used was one prepared by Mr. Randolph of this laboratory. Langbein[17] gives the composition of the bath as:-

Platinum chloride	0.245 oz.
Sodium phosphate	4.94 oz.
Ammonium phosphate	0.99 oz.
Sodium chloride	0.245 oz.
Borax	0.087 oz.

These were dissolved in six quarts of water and boiled for ten hours, the evaporating water being continually replaced. Before plating each piece was polished and carefully cleaned by repeated washings in dilute hydrochloric acid, then water, then alcohol to remove all grease, and finally rinsed thoroughly in distilled water. The platinum had to be deposited hot, so the beaker containing the solution was surrounded by nearly boiling water. The object was connected to the kathode and completely immersed in the bath. The anode was a piece of platinum

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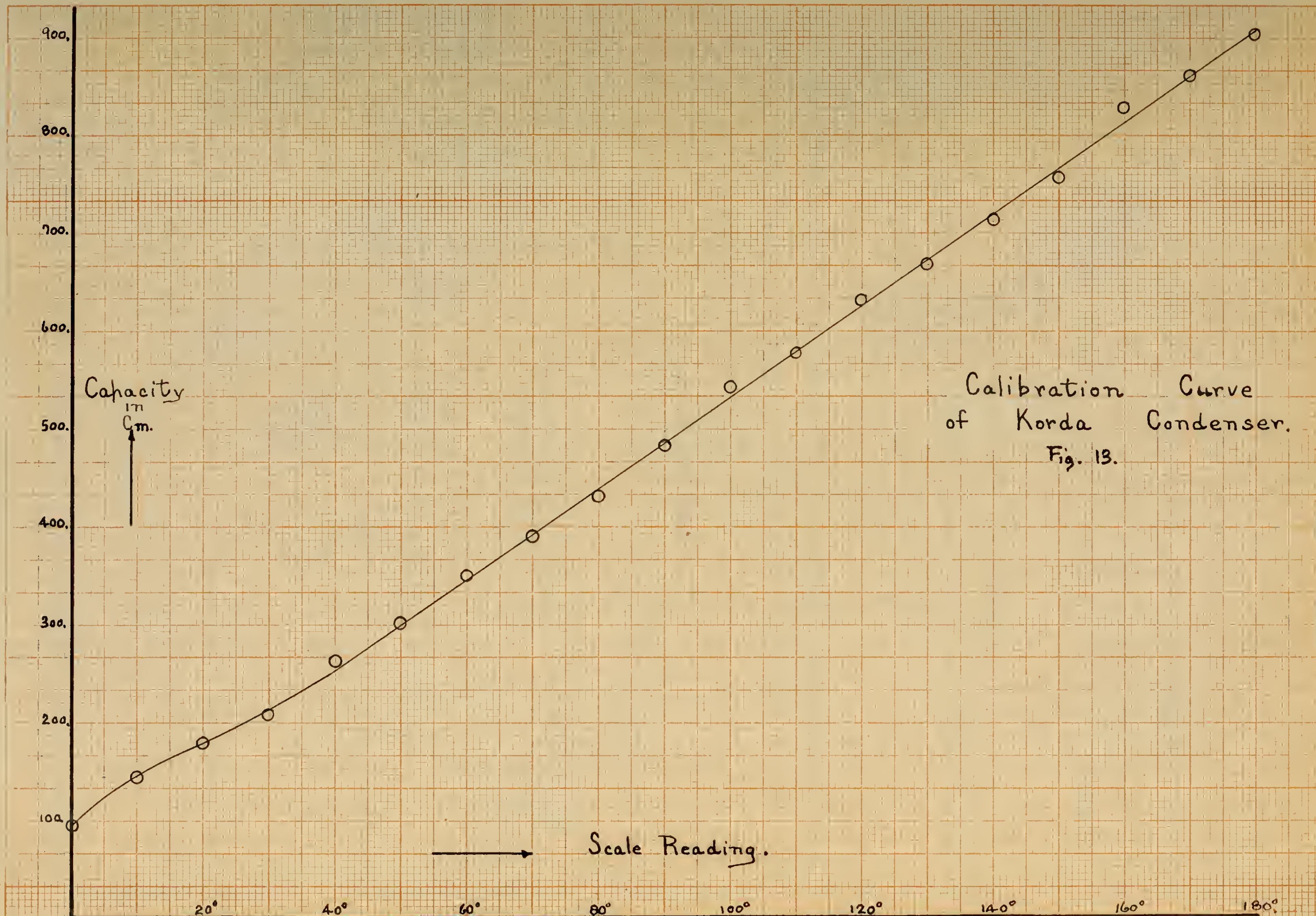
17 G. Langbein "Electrodeposition of Metals," translated by W.I. Brannt, 3rd. Edition, p. 320.



Calibration Curve of Korda Condenser.
Fig. 13.

foil placed symmetrically with respect to the piece to be plated. The current was obtained from a battery of storage cells consisting of two cells in series and two sets in parallel. This arrangement produced a copious evolution of gas at the anode. Each piece remained in the bath for about fifteen minutes. Then it was removed, polished, cleaned and the process repeated.

V DETERMINATION OF THE FREQUENCY

The frequency n of the oscillations in the secondary was determined by substituting in the formula

$$T = \frac{1}{n} = 2\pi\sqrt{LC} \tag{38}$$

when the secondary was in tune with the primary the variable Korda registered 969 cm. of capacity. L, the self inductance of the loop computed from Kirchhoff's[18] formula

$$L = 4\pi a\left(\log_\epsilon \frac{8a}{r} - 1.75\right) \tag{39}$$

was 473 cm. Then since (38) calls for inductance and capacity in Henries and Farads

$$n = \frac{1}{2\pi\sqrt{\frac{969 \cdot 473}{9} \cdot 10^{-20}}} = 7.03 \cdot 10^{7}$$

or the frequency is about seventy million.

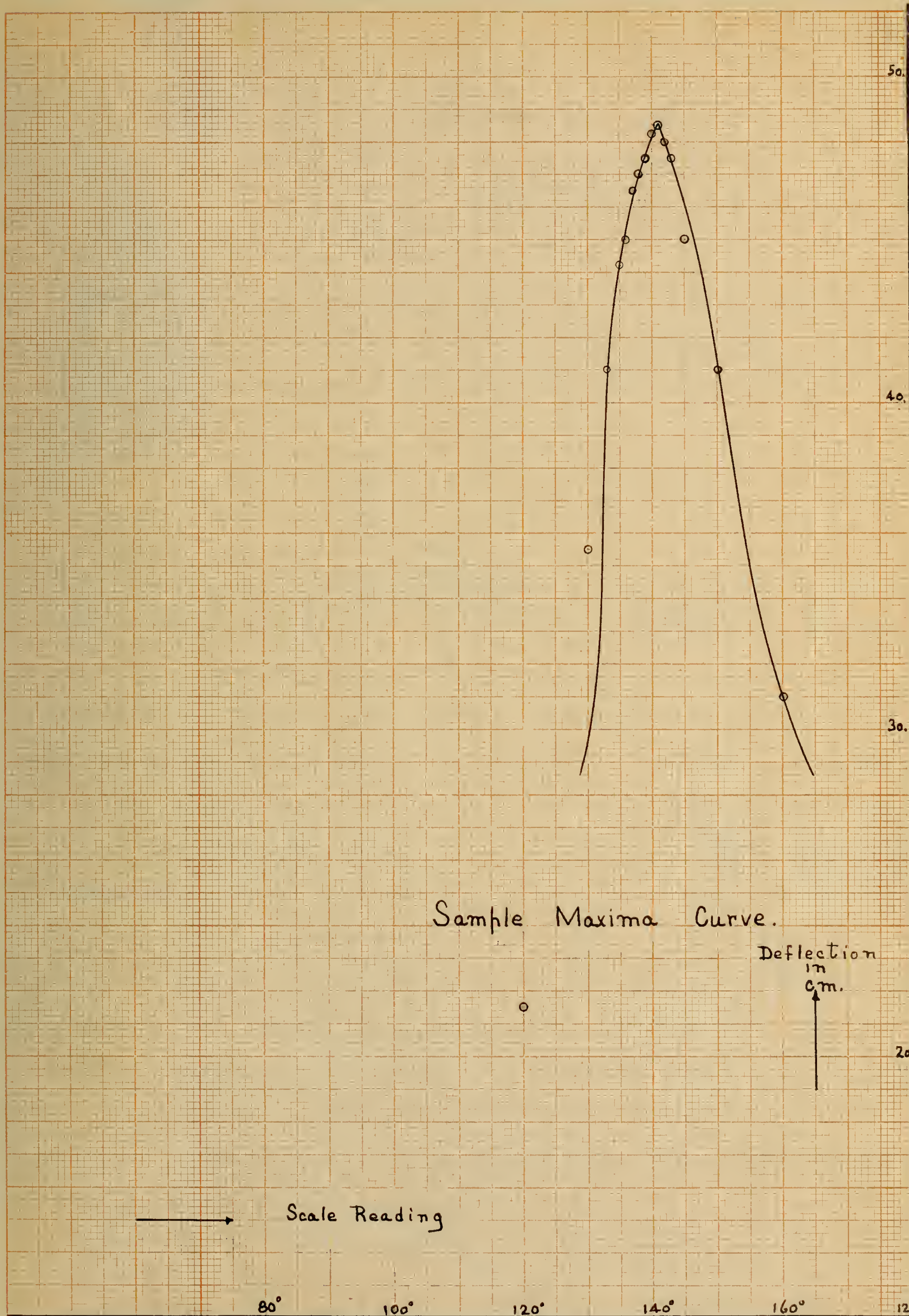
VI DISCUSSION OF THE METHOD AND THE ACCURACY

The final error in any method depends upon the errors made in determining each term of the working formula. This formula was

$$k = \frac{C' - C'''}{C_1} \tag{40}$$

The accuracy in each term of the numerator depends upon the accuracy of tuning. For each trial the apparatus was tuned several times and the average capacity taken. The following figures show how the several

18 See Bulletin U.S. Bureau of Standards, p. 55, (1908-09).


50.
40.
30.
20
12
Sample Maxima Curve.
Deflection in cm.
Scale Reading
80°
100°
120°
140°
160°

readings agreed.

169.5°	
170.0°	
171.0°	Mean 170.4
170.0°	Maximum possible error 0.9
171.0°	
171.0°	

The sharpness of the maxima can be seen from the curve on the following page. The denominator was determined from formula (37). Four independent determinations of this capacity gave numbers which were proportional to

48.6
48.7
48.6
48.6

The error in data as accurate as the above leads, in the case of a dielectric constant of about 3, to a maximum possible error of approximately 2.4%. Under favorable conditions maximum data could be obtained which was a little more accurate than the sample given. It seems reasonable, in the case of low dielectric constants, to estimate the maximum possible error as about 2%. The reading of C' seemed to change slightly from day to day, as if it depended to some extent upon the batteries to which the induction coil was connected. If this could be avoided, and it probably would be by using a quenched spark, C' could be determined a great many times and the mean taken as accurate. If this were done the maximum possible error would be about 1%. In these error estimations the calibration curve has been assumed correct. The electrometer was working perfectly when the calibration data was taken. The curve being an average of many points is probably more accurate than any one of the individual points.

Errors due to the liquid being conducting and thus decreasing

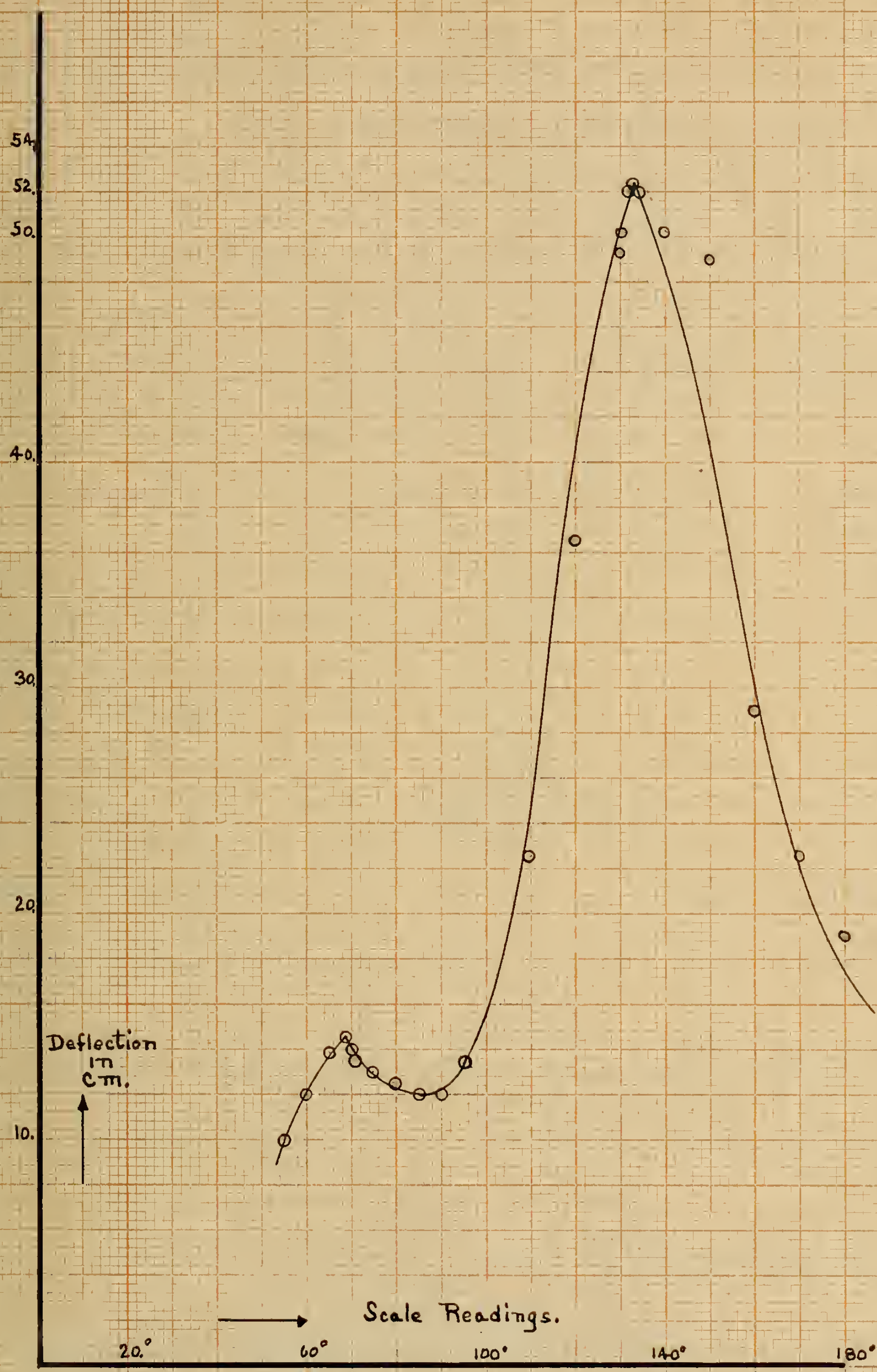
the maxima were avoided by placing a very large capacity (1 micro-farad) in series with the cone. This overcame conduction entirely and did not change the capacity of the cone, as has been shown by equation (17).

The maxima were kept large, never less than 50 cm., to insure their accurate determination. The intensity of the maxima could be regulated by advancing or withdrawing the secondary inductance from the primary coil. In general the coils were never closer than 5 cm.

The method is very sensitive to capacity changes. The near presence of the body would change the maximum points very noticeably. To avoid such errors all the apparatus except the induction coil and galvanometer was placed in a grounded metal box. So protected the maxima were independent of outside influences.

To measure substances of higher dielectric constant more capacity had to be put in the secondary and in order to tune with the primary the inductance of the primary had to be increased. It was found under these conditions that the maxima could not be determined as accurately as with the lower capacity. Thus this method fails to give accurate results for substances which have high dielectric constants.

The study of a sulphur slab brought out an interesting phenomena. In attempting to tune, two distinct maxima were observed. The first was small but very noticeable and the second large. The curve on the next page represents the data taken upon this slab. Overtones in electric resonance have been demonstrated. In this case the dielectric constant computed from the first weak maximum is the proper value for sulphur and the one computed from the large maximum is much too low. To justify taking the small maxima as the fundamental it must be assumed that the first harmonic is more intense than the fundamental.



Double Maxima shown by Sulphur.

Similar resonance phenomena are known in sound. Sulphur is the only substance which showed this double resonance.

VII STATEMENT OF RESULTS

Experiments have not been made with a great variety of substances but enough liquids and solids have been studied to show that the method is accurate and convenient. It is a very sensitive method when the substance has a low dielectric constant. The following table shows the results that have been obtained.

Substance	Dielectric Constant	Results By Other Observers
Kerosene	2.01	1.99 - 2.10
Turpentine	2.26	2.28
Cotton Seed Oil	2.97	3.00
Castor Oil	4.60	4.49 - 4.65
Alcohol 95%	26.0	23.0 to 26.3
Sulphur (‖ axis)	3.38	3.5 - 4.6
Paraffin	1.98	1.70 - 2.10

VIII SUMMARY

The maximum possible error for substances of low dielectric constant by this method is about 2%, but under favorable conditions the probable maximum possible error is about 1%.

High dielectric constants can not be accurately determined with the available capacities.

The necessary measurements can be easily and quickly taken.

Computation is a minimum, being simply a ratio.

The method is one that can be easily applied to the study of the variation of dielectric constants with temperature and pressure.

Only a small amount of the liquid to be studied is necessary -

about 20 c.c.

It seems that with a proper arrangement of capacities the method can be made sensitive for substances of higher dielectric constants. It is hoped that the method can be extended and improved, making one applicable for accurate study of the variation of capacity with frequency.

In conclusion the author wishes to express his appreciation to Professor A. P. Carman for his valuable advice and many suggestions so freely offered throughout this investigation.

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May, 1914